

# THE EXTENDED FINITE ELEMENT METHOD FOR TWO-PHASE FLOW

J. Chessa<sup>a</sup> and T. Belytschko<sup>b</sup>

<sup>a</sup>Department of Mechanical and Industrial Engineering  
The University of Texas at El Paso  
El Paso, TX 97712  
jfchessa@utep.edu

<sup>b</sup>Department of Mechanical Engineering  
Northwestern University  
Evanston, IL 60208  
t-belytschko@northwestern.edu

An extended finite element method (X-FEM) with arbitrary interior discontinuous gradients is applied to two-phase immiscible flow problems. The extended finite element method expands or “*extends*” the standard finite element basis so that arbitrary functions can be reproduced by the approximation [1,2]. We construct the extended finite element approximation so that it can capture the appropriate discontinuities in the velocity field at the interface without requiring the mesh to conform to the interface, eliminating the need for remeshing. Both no-slip and full slip interface conditions are considered. For the no-slip assumption the discontinuity in the derivative of the velocity field is introduced by an enrichment with an extended basis whose gradient is discontinuous across the interface. For the full slip assumption the discontinuity in the velocity field is introduced by an enrichment with an extended basis which is discontinuous in the tangential direction at the phase interface.

The equations for incompressible flow are solved by a fractional step method where the non self adjoint operators are stabilized by a characteristic Galerkin method [4]. The phase interfaces are tracked by level set functions which are discretized by the same finite element mesh and are updated via a stabilized conservation law [3].

The method is applied to several examples including the falling and merger of axisymmetric drops, free surface flows, shear flows, bubbles rising to a free surface as well as surface tension effects.

KEYWORDS: enriched finite element, X-FEM, level set, two-phase flow.

## References

- [1] T. Belytschko and T. Black. Elastic crack growth in finite elements with minimal remeshing. *International Journal of Numerical Methods in Engineering*, 45(5):601–620, 1999.
- [2] N. Moës, J. Dolbow, and T. Belytschko. A finite element method for crack growth without remeshing. *International Journal of Numerical Methods in Engineering*, 46:131–150, 1999.
- [3] S. Osher and J. A. Sethian. Propagation of fronts with curvature based speed: algorithms based on Hamilton-Jacobi formulations. *Journal of Computational Physics*, 79:12, 1988.
- [4] O.C. Zienkiewicz and R. Codina. A general algorithm for compressible and incompressible flow, Part I. The split characteristic based scheme. *International Journal of Numerical Methods in Fluids*, 20:869–85, 1995.